Course name : **Fractional calculus and its applications**		
Type of study: **Mathematics**	Type of study: **Full-time**	Examination: **Assignment**
Course characteristics: **Facultative**	Level: **Second (M.Sc.)**	Year: **Full year**
Type of classes: **lectures, tutorials**	Hours per week: **2 lect, 2 tut**	ECTS points: **6 ECTS**

COURSE GUIDE

AIMS

A1. Making the students familiar with the elements of non-integer order calculus.

A2. Developing students' practical skills to solve fractional differential equations (FDEs) and to interpret solutions.

A3. To acquaint the students with the applications of the fractional calculus and the fractional differential equations theory.

PREREQUISITES

1. Course of the mathematical analysis.
2. Course of the complex analysis.
3. Ability to use different sources of information.
4. Ability to work independently and in a group.
5. Ability to correctly interpret and present their own activities.

LEARNING OUTCOMES

EE 1 – student is familiar with the basics of fractional calculus.

EE 2 – student is able to solve different types of non-integer order differential equations.

EE 3 – student has basic knowledge about applications of fractional calculus.

CONTENT

Lectures	Hours
Lect. 1 - Course introduction. Gamma and beta functions.	**2**
Lect. 2 - Mittag-Leffler functions. Wright function.	**2**
Lect. 3 – Fractional integrals for functions of one variable in a bounded and unbounded domain. Definitions, examples and properties.	**2**
Lect. 4, Lect. 5 – Left and right fractional derivatives. Caputo, Riemann-Liouville and Liouville definitions. Properties.	**4**
Lect. 6 – Composition rules for integrals and derivatives.	**2**
Lect. 7 – Fractional differential nonlinear equations – existence and uniqueness results.	**2**
Lect. 8 – Linear differential equations of non-integer order.	**2**
Lect. 9 – Basset's problem.	**2**
Lect. 10 – Fractional Sturm-Liouville problem – variational formulation.	**2**

Lect. 11, Lect. 12 – Introduction to Laplace and Fourier transforms.	**4**
Lect. 13 – Cauchy problems for time-fractional partial differential equations.	**2**
Lect. 14 – Anomalous diffusion – subdiffusion.	**2**
Lect. 15 – Anomalous diffusion – superdiffusion.	**2**
TUTORIALS	Hours
Tut. 1 – Gamma and beta functions.	**2**
Tut. 2 – Mittag-Leffler and Wright functions.	**2**
Tut. 3 – Riemann-Liouville integrals – examples of integration.	**2**
Tut. 4, Tut. 5 – Fractional differentiation – examples.	**4**
Tut. 6 – Properties of fractional derivatives.	**2**
Tut. 7 – Application of the properties of fractional operators in transformation of fractional differential equations (FDEs).	**2**
Tut. 8 – Fixed point theorem as a method of solving FDE.	**2**
Tut. 9, Tut. 10 – Linear FDEs with constant coefficients.	**4**
Tut. 11 – Laplace and Fourier transforms – examples.	**2**
Tut. 12 – Laplace transform - application in solving linear differential equations of integer and non-integer order.	**2**
Tut. 13, Tut. 14 – Laplace-Fourier transform for partial FDEs.	**4**
Tut. 15 - Test.	**2**

TEACHING TOOLS

1. – lectures using multimedia presentations
2. – blackboard and chalk or whiteboards and pens

LITERATURE

Lecture notes.
A.A. Kilbas, H.M. Srivastava, J.J. Trujillo *Theory and Applications of Fractional Differential Equations*, Mathematics Studies 204. North-Holland 2006
B.J. West, M. Bologna, P. Grigolini, *Physics of Fractal Operators*, Springer 2003
Lokenath Debnath, Dambaru Bhatta, *Integral Transforms and Their Applications,* Chapman and Hall/CRC 2006
M. Klimek, *On Solutions of Linear Fractional Differential Equations of a Variational Type*, The Publishing Office of the Czestochowa University of Technology 2009

TEACHERS

1. **Prof. Malgorzata Klimek, mklimek@im.pcz.pl**
2. **Dr Tomasz Blaszczyk, tomasz.blaszczyk@im.pcz.pl**

ADDITIONAL NOTES

Links to course unit teaching materials can be found on the http://www.pcz.pl/english/ects-subjects website for current students.