Code	I.3.
Course Title (English)	Mathematics I
Course Title (Polish)	Matematyka I
Credits	6 ECTS

Language of instruction **English**

Compulsory for Profile: Computer Modelling and Simulation (CMS), Intelligent Energy (IE), Biotechnology for Environmental Protection (BI), Business and Technology (BT)

Type of studies BSc studies

Unit running the programme Institute of Mathematics and Computer Science

Course coordinator *and academic teachers* **M. Klimek Ph. D**., M. Klimek Ph. D. (Lec.), Z. Domański Ph. D. (Tut.)

Form of classes and number of hours

Semester	Lec.	Tut.	Lab.	Proj.	Sem.	Credit points
autumn	45	30	-	-	-	6

Learning outcomes

Acquiring knowledge of basic notions of linear algebra. Extending knowledge of analytic geometry on plane to the analytic geometry in three-dimensional space. Acquiring and/or improving knowledge of differential and integral calculus of functions of one variable. Learning mathematical skills that can be applied to other technical and economic courses.

Linear algebra

Specifically student should be able to:

- distinguish between vector and scalar quantities
- use vector notation and represent a vector pictorially
- carry out scalar multiplication of a vector and represent it pictorially
- determine the unit vector in a specified direction
- represent a vector in component form for a given basis
- understand what is meant by a matrix and recall the basic terms associated with matrices
- obtain the transpose of a matrix
- determine any scalar multiple of a matrix
- recognize when two matrices can be added (or multiplied) and find, where possible, their sum (product)
- calculate the determinant and use the elementary properties of determinants in their evaluation
- state the criterion for a square matrix to have an inverse and determine the inverse of a matrix
- calculate the rank of a matrix
- represent a system of linear equations in matrix form
- understand how the general solution of an inhomogeneous linear system of *m* equations in *n* unknowns is obtained from the solution of the homogeneous system and a particular solution
- recognize the different possibilities for the solution of a system of linear equations
- understand how and why the rank of the coefficient matrix and the augmented matrix of a linear system can be used to analyze its solution
- use the inverse matrix to find the solution of the system of simultaneous linear equations

Analytic geometry

Specifically student should be able to:

- recognize and use the standard equation of a straight line in 3D
- recognize and use the standard equation of a plane
- find the angle between two straight lines
- find where two straight lines intersect
- find the angle between two planes
- find the intersection line of two planes
- find the intersection of a line and a plane
- find the angle between a line and a plane
- calculate the distance between two points, a point and a line, a point and a plane
- calculate the distance between two lines, a line and a plane, two planes
- recognize and use the standard equation of a singular quadratic surface (cylindrical, conical)
- recognize and use the standard equation of a regular quadratic surface (ellipsoid, paraboloid, hyperboloid)
- solve simple problems in geometry using vectors
- define the scalar (vector) product of two vectors and use it in simple applications
understand the geometric interpretation of the scalar (vector) product
define the scalar triple product of three vectors and use it in simple applications
- understand the geometric interpretation of the scalar triple product.

Functions of one variable : differential and integral calculus

Specifically student should be able to:

- understand convergence and divergence of a sequence
- know what is meant by a partial sum
- apply simple tests for convergence of a series
- obtain the partial fractions of a rational function, including cases where the denominator has a repeated linear factor or an irreducible quadratic factor
- define and recognize an odd function and an even function
- understand the properties 'concave' and 'convex'
- understand the concepts of continuity and smoothness
- differentiate inverse and composite functions
- differentiate functions defined implicitly
- differentiate functions defined parametrically
- locate the extrema of a function
- locate any points of inflection of a function
- find greatest and least values of physical quantities
- find the tangent and quadratic approximations to a function
- understand the idea of radius of convergence of a power series
- recognize Maclaurin series for standard functions
- understand how Maclaurin series generalise to Taylor series
- obtain definite and indefinite integrals of rational functions in partial fraction form
- apply the method of integration by parts to indefinite and definite integrals
- use the method of substitution on indefinite and definite integrals
- solve practical problems which require the evaluation of an integral
- recognize and calculate improper integrals
- find the length of part of a plane curve
- find the curved surface area of a solid of revolution
- find the first and second moments of a plane area about an axis
- find the centroid of a plane area and of a solid of revolution.

Prerequisites

Secondary school knowledge of mathematics is essential. We expect sound understanding of arithmetic, classical algebra, geometry, trigonometry, elementary differential calculus and probability theory. Specifically,

1) real numbers, algebraic operations, graphical and algebraic solutions of equations and inequalities;
2) elementary functions (algebraic and transcendental) – their graphs, properties as well as equations and inequalities involving them;
3) geometry – Euclidean geometry of a triangle and simple polygons, plane analytic geometry (straight line and conics);
4) introduction to differential calculus – sequences, limits of functions, concept of the derivative and its application to find slope of a curve;

Course description

LECTURE

Vector space: addition and scalar multiplication operations. Linear dependence of vectors. Basis and dimension of a vector space.

Matrices: matrix as a function. Addition and scalar multiplication. Vector space with matrices as a vectors. Matrix product. Properties of matrix multiplication. Matrix form of the system of linear algebraic equations. Special matrices. Diagonal and identity matrix. Transpose of a matrix. Symmetric, skew symmetric and upper triangular matrix.

Determinants: definition, minor, cofactor. Cofactor expansion by row and by the column of the determinant. Properties of the determinants. .Elementary row operations. Rank of the matrix.

Homogeneous and nonhomogeneous system of linear equations.
Augmented matrix. Procedure for solving the system. Existence and uniqueness of the solution. Matrix inverse. Singularity of the matrix. A determinant formula for the inverse of a matrix. Cramer`s Rule. Kronecker Capelli theorem.

Analytic geometry.
Dot product. Direction angles. Direction cosines. The cross product. Scalar triple product. Properties. Equation of a line. Vector equation, parametric equations, symmetric equations, direction numbers. Skew lines. Equation of a plane. Normal vector. Vector, scalar and linear equation of a plane. Parallel planes. Formula for the distance from a point to a plane.

Quadric surfaces.
Traces. Ellipsoid. Hyperboloid of one sheet. Hyperboloid of two sheets. Cone. Elliptic paraboloid. Hyperbolic paraboloid. Quadric cylinders. Elliptic cylinder. Parabolic cylinder. Identification of the surface.

Revision of elementary functions: graphs, properties. Composition of functions. Inverse functions including inverse trigonometric functions.

Number sequences: convergent and divergent sequences as well as their properties, Euler's number, indeterminate symbols.

Infinite series: series with positive terms, tests for convergence, absolutely and conditionally convergent series, rearrangements of terms of conditionally and of absolutely convergent series.

Differential calculus: limit of a function, continuous functions (definition and properties), derivatives (rules of differentiation), Rolle's theorem and the Mean-Value theorem, higher order derivatives, Taylor's and Maclaurin's formulae, maximum and minimum problems, convexity and inflection points, l'Hopital's rule, asymptotes of a curve, curve sketching.

Functional series: uniform and pointwise convergence, tests for convergence, properties of the uniformly convergent series, Power series, expanding functions into power series.

Integral calculus: antiderivatives, indefinite integral, techniques of integration, definite integrals (definition, properties, necessary and sufficient conditions for integrability, Newton-Leibniz theorem), improper integrals, application of integral calculus.

TUTORIALS: see lecture content

LABORATORY

Not applicable

PROJECT
Not applicable

SEMINAR

Not applicable

Form of assessment

Basic reference materials

Other reference materials

e-mail of the course coordinator and academic teachers	klimek@imi.pcz.czest.pl zdomanski@imi.pcz.czest.pl
Average student workload (teaching hours + individ)	5 hours of teaching hours + 3 hours of individual work per week
Remarks:	
Updated on: 04.04.2012	